Appendix B

Final Classification Report

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CLASSIFICATION OF INL VEGETATION DATA

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STEP 1: FINDING THE BEST CLASSIFICATION STRATEGY FOR THE INL DATA

METHODS

In my initial work I have tried to find the best possible classification method given the general cluster structure of the INL data. The data consisted of 314 plots and 111 plant species. Eight classification methods were compared: **(1)** average linkage (Sokal and Michener 1958), **(2)** centroid linkage (Sokal and Michener 1958), **(3)** complete linkage (McQuitty 1960), **(4)** flexible β = -0.25 (Lance and Williams 1967), **(5)** k-means analysis (MacQueen 1967), **(6)** partitioning around medoids, i.e., PAM (Kauffman and Rousseeuw 1990), **(7)** single linkage (Sneath 1957), and **(8)** variance minimization linkage, i.e., Ward's method (Ward 1963). PAM and k-means analysis are non-hierarchical methods while the other six are hierarchical agglomerative methods. Each k-means classification was the lowest sum of squares solution from 100 randomized starts. Steinhaus/Bray-Curtis dissimilarity (Bray and Curtis 1957) was used to quantify resemblance of sites for all methods except for k-means analysis (where Euclidean distance was used). Bray-Curtis dissimilarity generally outperforms Euclidean distance with typically sparse (few non-zero entries) vegetation datasets which may contain many plots with nothing in common (Beals 1984; McCune and Grace 2002). Over 90% of the cells in the INL data matrix contained zero entries. Within-dendrogram distances for hierarchical classifications were measured with Wishart's objective function (Wishart 1969) which prevents reversals (McCune and Grace 2002). Hierarchical agglomerative classifications were created using PC-ORD (McCune and Mefford 1999). Nonhierarchical classifications were created using the base and cluster (Maechler et al. 2005) libraries in R (R-core development team 2008).

The eight methods were compared using six classification evaluators: **(1)** indicator species analysis (ISA) number of significant indicators (Dufrêne and Legendre 1997; McCune and Grace 2002), **(2)** ISA average *p*-value (Dufrêne and Legendre 1997; McCune and Grace 2002), **(3)** C-index (Hubert and Levin 1976), **(4)** average silhouette width, i.e., ASW (Rousseeuw 1987), **(5)** point biserial correlation, i.e., PBC (Brogden 1949), and **(6)** partition analysis ratio, i.e., PARTANA (Roberts 2005, Aho et al. 2006a). For further detail on these procedures see Aho et al. (2008). All evaluators were programmed by Aho et al. (2006a) using the R language except for ASW which exists in the R-library cluster (Maechler et al. 2005).

The evaluator scores of the eight classification methods were compared with respect to their 49 simplest clustering solutions (i.e., 2 to 50 clusters) of the INL data. Comparisons of methods were made for each evaluator. Distributions of residuals from ANOVAs which compared methods were highly non-normal necessitating a non-parametric approach. Because classification methods were blocked by number of clusters, Friedman's method for non-parametric repeated measures (blocking without replication) was used (Table 1.1). The tested hypotheses were:

H_0: $\tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 = \tau_6 = \tau_7 = \tau_8 = 0$

H_A: at least one $\tau_i \neq 0$.

Where τ_i is the *i*th treatment effect (i = 1, 2,…, 8).

Note that the Freidman's test statistic asymptotically approaches a χ^2 distribution with *r* - 1 (8 - 1) degrees of freedom (Table 1.1). Friedman's test was conducted using the library coin in R (Hothorn et al. 2008).

Classification methods were also compared to each other in a non-parametric pairwise fashion adjusted for simultaneous inference (Table 1.1). These comparisons use the Friedman's pooled variances and are based on the method of Kutner et al. (2005, pgs. 1138-1139).

The tested hypotheses were:

H_0: $\tau_i = \tau_i'$.

H_A: $\tau_i \neq \tau_i'$.

Where τ_i is the *i*th treatment effect (i = 1, 2,…, 8).

RESULTS

Classifications of the eight methods differed significantly from the perspective of all six evaluators ($df = 7$, $131.4 \leq \chi^2 \leq 249.5$, $p < 2.2\times10^{-16}$; Table 1.1). Flexible β = -0.25 created the strongest classifications. It had the highest evaluator score for all six evaluators (Table 1.1). Ward's method classifications were also effective. Ward's had the highest evaluator score for five of the evaluators (Table 1.1). The best non-hierarchical method was PAM which had three highest evaluator scores (Table 1.1). In contrast, single linkage and *k*-means analysis created the weakest classifications. These methods did not have the highest evaluator score for any of the evaluators, and had the largest number of lowest evaluator scores (Table 1.1). Note that several methods can tie for best (or worst) method if they are statistically indistinguishable (Table 1.1).

DISCUSSION/CONCLUSIONS

In general, methods which created spherical clusters (i.e., average linkage, flexible β = -0.25, complete linkage, and PAM, Ward) were favored by the evaluators. This was true for both the geometric evaluators (which are predisposed to favoring spherical clusters) and non-geometric evaluators (i.e., the indicator species analysis methods) which are not predisposed to favor a particular type of cluster geometry. As a result, it appears as if a spherical cluster interpretation of the INL data (as opposed to a linear cluster interpretation) is the most valid one. In particular, I recommend using flexible β = -0.25 as the classification method for the INL data.

Table 1.1. Comparison of the classification efficacy of eight classification methods with respect to six classification evaluators.

In the upper part of the table classification methods are statically compared using Friedman's non-parametric repeated measures analysis.

In the lower part of the table classification methods are individually compared with respect to each evaluator. In particular, medians, 95% CI for medians, and pairwise test results are summarized. Ninety-five percent confidence intervals for medians were computed using the method of McGill *et al.* (1978, p. 16). Methods designated with the same letter are not significantly different in pairwise comparisons using $\alpha = 0.05$. Methods with the highest evaluator score(s) were assigned A, the next highest were assigned B, and so on. Number of highest scores indicate the number of times a method performs best with respect to the six evaluators (i.e. the number of "A's" for a particular method). Low scores indicate the number of times a classification method did worst with respect to the six evaluators (i.e. the number of times a method was assigned a letter furthest from A).

	ISA # of sig. inds	ISA avg. *p*-value	C-Index	ASW	PBC	PARTANA		
Freidman's Test Summary								
χ^2	249.4732	257.6871	131.3722	211.8815	207.2875	176.7709		
p-value	$< 2.2 \times 10^{-16}$	$< 2.2 \times 10^{-16}$	$< 2.2 \times 10^{-16}$	$< 2.2 \times 10^{-16}$	$< 2.2 \times 10^{-16}$	$< 2.2 \times 10^{-16}$		
df	7	7	7	7	7	7		
Median ± 95% CI for median, and pairwise test results							**Number of Highest Scores**	**Number of Lowest Scores**
Average	33 ± 2.26 B	0.68 ± 0.028 B,C	0.89 ± 0.02 A	0.22 ± 0.016 A,B	0.55 ± 0.011 A	3.46 ± 0.196 A,B	4	0
Centroid	26 ± 0.68 C,D	0.66 ± 0.035 C,D	0.83 ± 0.019 A,B	0.08 ± 0.029 D,E	0.43 ± 0.039 C	2.85 ± 0.176 C	1	3
Complete	33 ± 3.39 B	0.67 ± 0.019 B	0.86 ± 0.024 A,B	0.2 ± 0.011 B,C	0.53 ± 0.014 A	3.42 ± 0.197 B,C,D	2	1
Flexible	40 ± 0.68 A	0.73 ± 0.003 A	0.87 ± 0.027 A	0.22 ± 0.012 A	0.51 ± 0.01 A,B,C	3.55 ± 0.184 A	6	0
K-means	31 ± 1.58 B,C	0.66 ± 0.006 B,C	0.71 ± 0.023 D	0.15 ± 0.006 C,D	0.37 ± 0.015 D	2.75 ± 0.209 D	0	4
PAM	39 ± 0.68 A	0.73 ± 0.005 A	0.84 ± 0.014 B	0.18 ± 0.005 B	0.46 ± 0.02 B,C	3.46 ± 0.125 A,B,C	3	0
Single	12 ± 1.35 D	0.61 ± 0.017 D	0.77 ± 0.025 C	-0.1 ± 0.017 E	0.25 ± 0.013 D	1.9 ± 0.08 E	0	6
Ward	42 ± 0.9 A	0.75 ± 0.003 A	0.86 ± 0.021 A,B	0.21 ± 0.007 A,B	0.48 ± 0.015 B	3.52 ± 0.162 A	5	0

STEP 2: FINDING THE OPTIMAL NUMBER OF CLUSTERS IN THE INL DATA

METHODS

In step one, eight classification methods were compared and flexible β = -0.25 (Lance and Williams 1967) was found to be the best overall method for the INL data. In step two, I found the optimal numbers of clusters in the flexible β = -0.25 classification.

The same six evaluators used in step one were used as optimality criteria. The geometric evaluators (ASW, C-index, PARTANA, and PBC) index classification effectiveness based on cluster compactness and distinctness in multivariate space (cf. Dale 1991). The non-geometric evaluators (ISA number of significant indicators, and ISA average *p*-value) measure classification effectiveness with respect to indicator species. For instance, a clustering solution in which a species occurs predominantly in one cluster while being absent from others indicates a "real" cluster structure from the perspective of that species (Aho et al. 2008).

Forty-nine possible classification solutions were examined (2 to 50 clusters). Evaluator scores were centered and scaled (Eq. 2.1) to allow all evaluators to be displayed in a single figure.

$$E_{jk} = \frac{X_{jk} - \overline{X}_k}{S_k} \qquad (2.1)$$

Where E_{jk} is the standardized evaluator score of the *j*th cluster from the *k*th evaluator (j = 2, 3,..., 50), (k = 1, 2, …,6); X_{jk} is the raw evaluator score of the *j*th cluster from the *k*th evaluator; $\overline{X}_k$ is the *k*th evaluator sample mean; and S_k is the *k*th evaluator sample standard deviation.

In addition, because some evaluators always tend to increase/decrease with number of clusters (Aho 2006a), linear models were used to identify and account for this trend (Eq. 2.2). Residuals (Eq. 2.3) from these models (now standardized values with respect to linear trends) were used as indicators of classification efficacy (See Fig. 2.1).

$$\hat{Y}_{jk} = X_{jk} b_1 + b_0 \qquad (2.2)$$

Where $\hat{Y}_{jk}$ is the predicted evaluator score of the *j*th cluster from the *k*th evaluator (j = 2, 3,..., 50), (k = 1, 2, …,6); X_{jk} is *j*th cluster number from the *k*th evaluator; and b_1 and b_0 are least squares estimates for the linear regression slope and *Y*-intercept.

$$e_{jk} = Y_{jk} - \hat{Y}_{jk} \qquad (2.3)$$

Where Y_{jk} are observed values and $\hat{Y}_{jk}$ are fitted values from the linear regression model in Eq. 2.2.

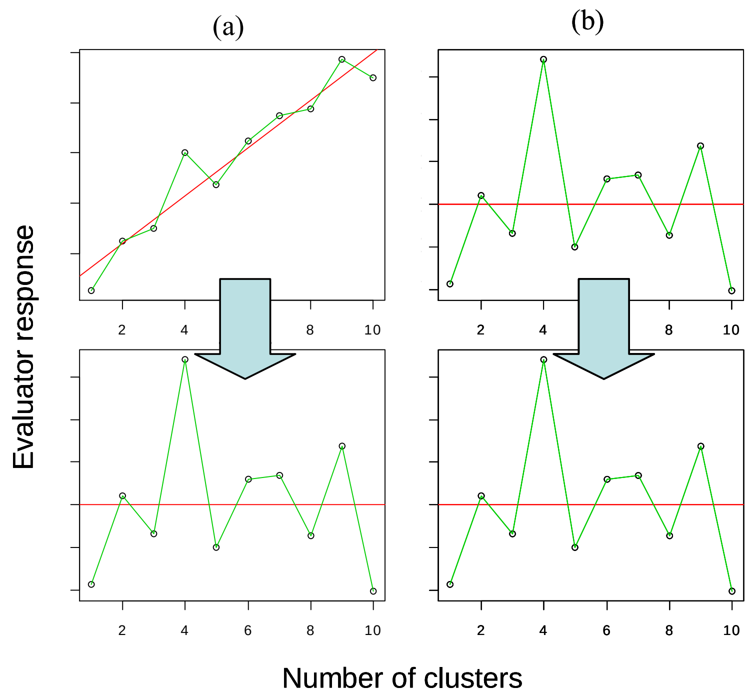


Figure 2.1. Converting evaluator scores (upper figures) to residuals from linear models (lower figures). This is demonstrated for evaluators with possible linear artifact (a), and without linear artifact (b). Note that while predicted optima for the evaluator with linear artifact (a) are radically adjusted, (b) is unchanged (from Aho 2006a).

RESULTS

All six evaluators found solutions with only two or three clusters to be the worst possible solutions (Figure 2.2, Table 2.1). The non-geometric ISA evaluators found 8- and 12-cluster solutions to be optimal while the geometric evaluators found a higher number of clusters 17-22 to be optimal (Table 2.1).

Averaging the scores of all six evaluators, the best solution was 12 clusters, although solutions around twenty clusters were also favored (Figure 2.3). Classifications with fewer than eight clusters and more than 30 clusters were found to be particularly poor (Figure 2.3).

NMDS ordinations (Kruskal and Wish 1978) were used to demonstrate the spread of sites in multivariate species space and the relationship of the clusters to each other (Figures. 2.4-2.6).

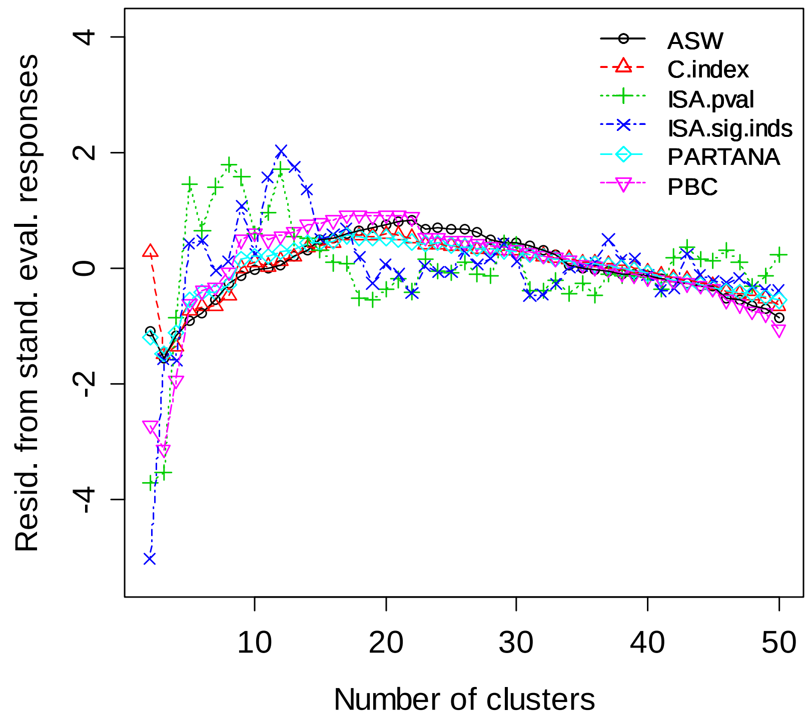


Figure 2.2. Scores of six evaluators with respect to 49 clustering solutions (two to 50 clusters) of the INL vegetation data.

Table 2.1. Tabular summary of Figure 2.2

	best solution	worst solution
ASW	22	3
C-index	20	3
ISA avg. *p*-value	8	2
ISA # of sig. inds	12	2
PARTANA	17	3
PBC	17	3

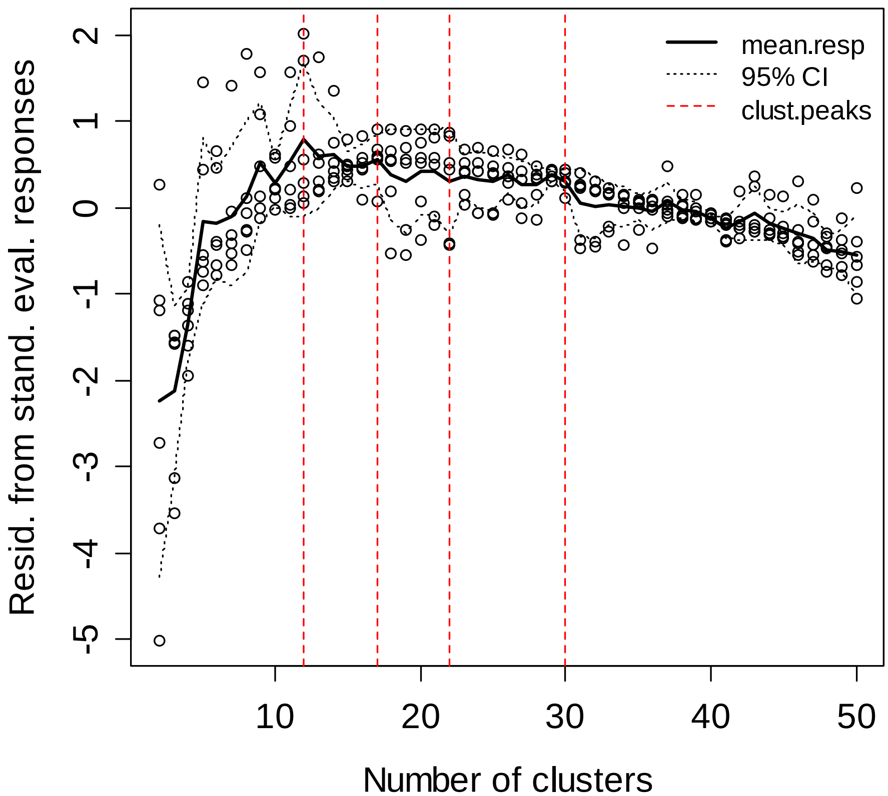
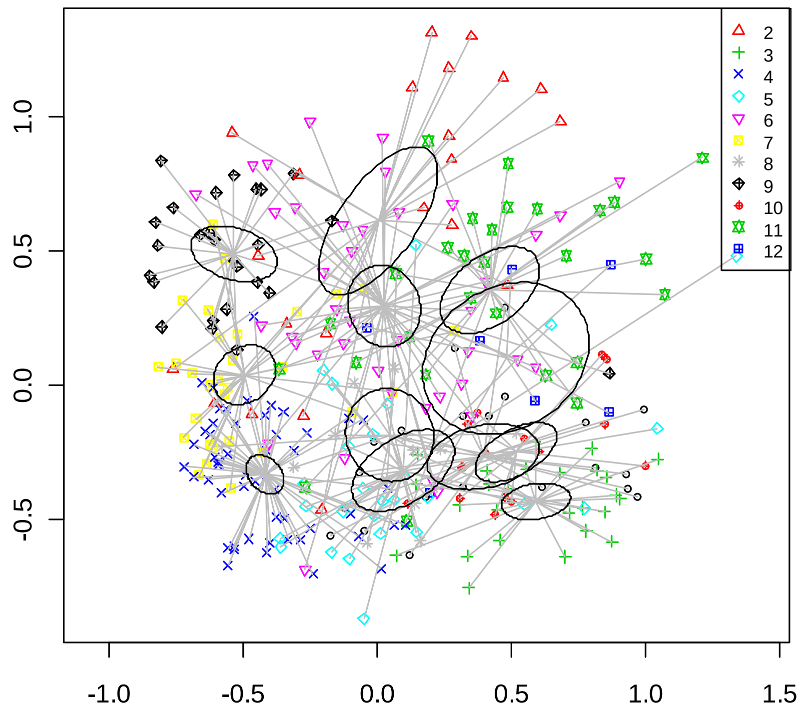


Figure 2.3. Scores of six evaluators with respect to 49 clustering solutions (two to 50 clusters) of the INL vegetation data. Averages shown with a solid black line. Ninety-five percent confidence intervals around the mean shown with black dashed lines. Red dashed lines are at 12, 17, 22, and 30 clusters

(a)



(b)

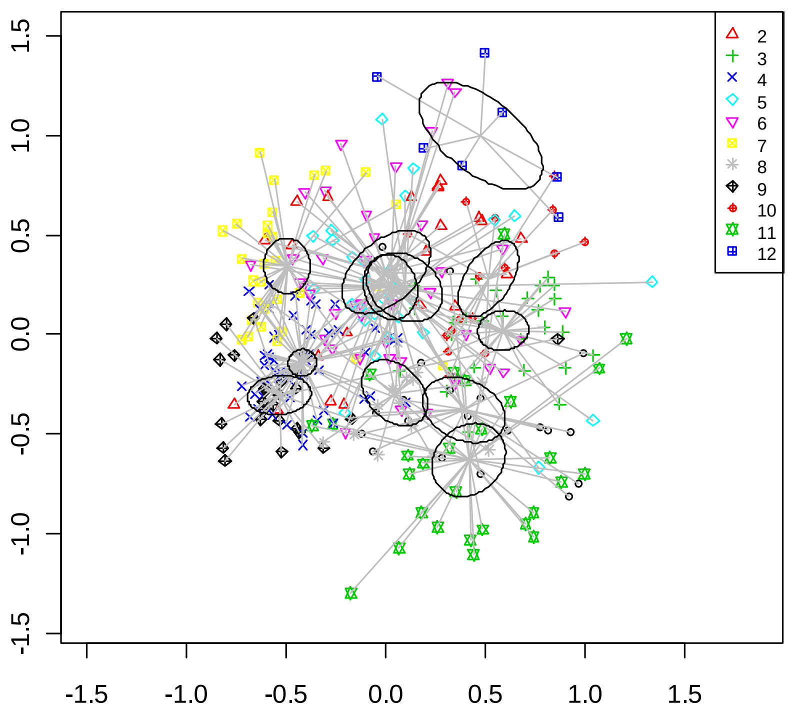
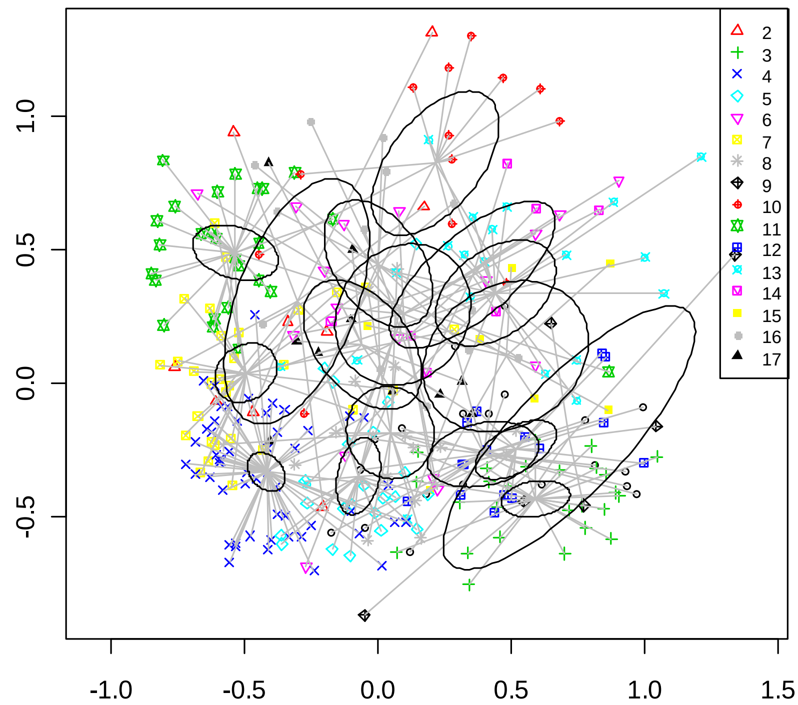


Figure 2.4. NMDS ordination of the INL vegetation data. Twelve cluster solution overlaid. (a) Dimensions 1 and 2; (b) dimensions 1 and 3. Final stress for 3-dimensional solution = 21.5. Steinhaus dissimilarity used to create distance matrix. Ellipses are 95% confidence intervals around cluster centroids.

(a)



(b)

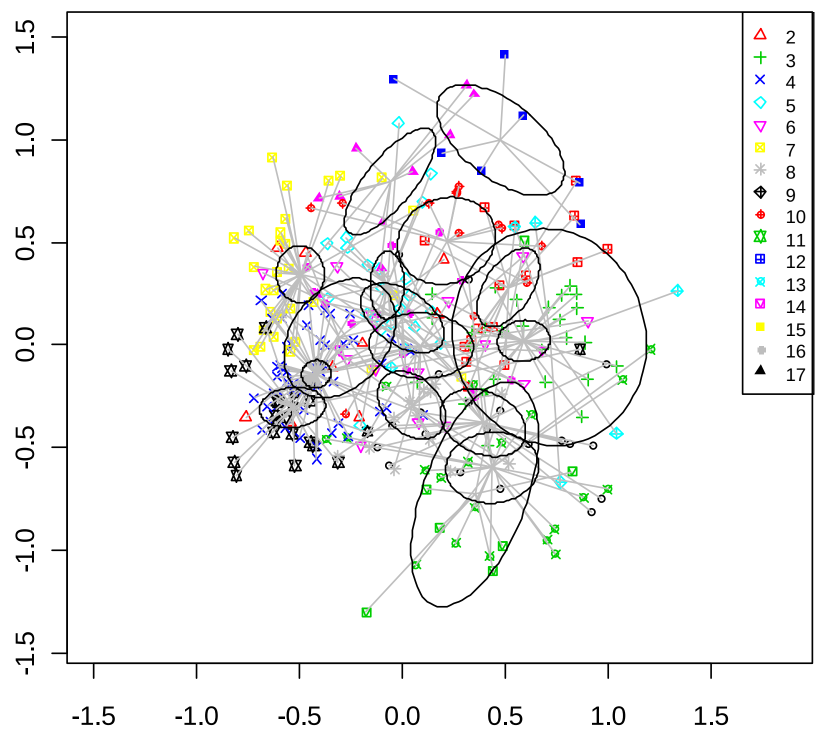
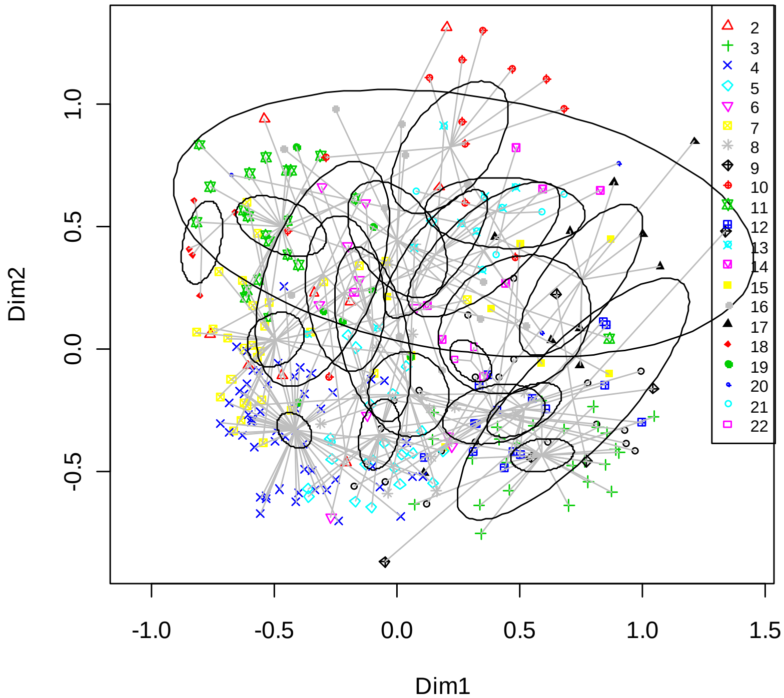


Figure 2.4. NMDS ordination of the INL vegetation data. Seventeen-cluster solution overlaid. (a) Dimensions 1 and 2; (b) dimensions 1 and 3. Final stress for 3-dimensional solution = 21.5. Steinhaus dissimilarity used to create distance matrix. Ellipses are 95% confidence intervals around cluster centroids.

(a)



(b)

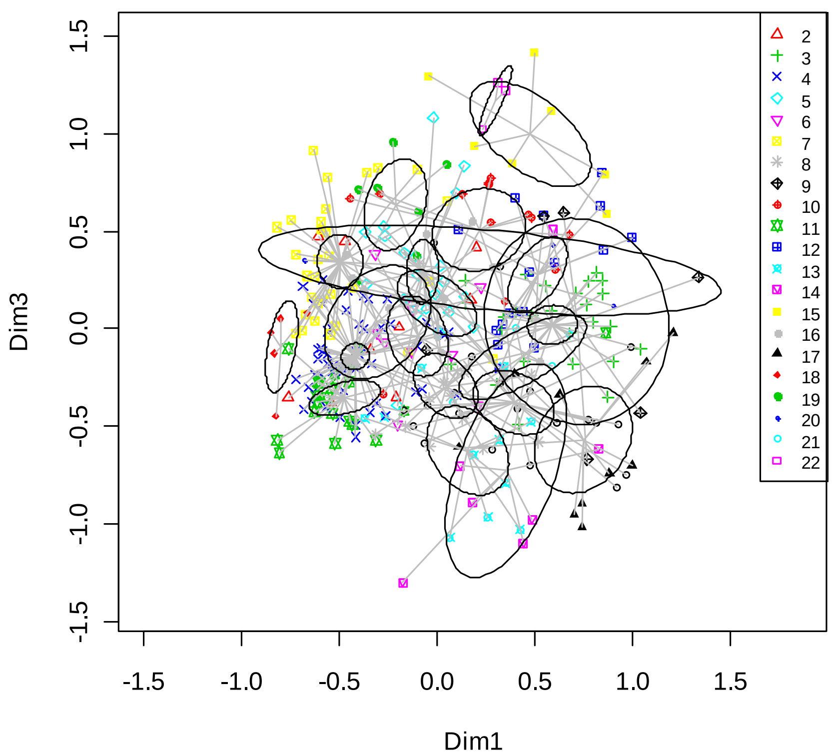


Figure 2.4. NMDS ordination of the INL vegetation data. Twenty-two cluster solution overlaid. (a) Dimensions 1 and 2; (b) dimensions 1 and 3. Final stress for 3-dimensional solution = 21.5. Steinhaus dissimilarity used to create distance matrix. Ellipses are 95% confidence intervals around cluster centroids.

STEP 3: CLASSIFICATION DESCRIPTION

METHODS

In this step, optimal classifications with 12 and 22 clusters are described. A 12-cluster solution was favored by non-geometric evaluators, particularly ISA average *p*-value (Fig. 2.2, Table 2.1), while a solution around 22 clusters was favored by a number of the geometric evaluators, particularly average silhouette width (Fig. 2.1, Table 2.1).

Classification descriptions follow the method used by Aho et al. (2005) in consulting work with the Montana Dept. of Environmental Quality. This work utilized relevé tables which described both cover and constancy of plant species within communities. Cover is the average percent ground cover of a species within a community. Constancy is defined is the percentage of time a species occurs in a particular community. Thus for species j within community k (which was sampled with n_k sites) constancy is calculated as:

$$Const_{jk} = \frac{O_{jk}}{n_k} \qquad (3.1)$$

Where O_{jk} is the number of sites within community k that species j occurs in and n_k is the number of sites which describe community k.

Aho (2006b, pg. 371) introduced a function to sort species based on a meaningful ordering of sites or groups of sites. In particular, in a summary relevé table, where sites are grouped based on the similarity of their species composition, species can be sorted with respect to their fidelity to the groups. Fidelity is the percentage of times a species occurs in a particular community compared to its occurrences in all other communities (Eq. 3.2). For species j within community k (sampled with n_k sites) fidelity is calculated as:

$$Fidel_{jk} = \frac{O_{jk}}{\sum_{k=1}^{g} O_{jk}} \qquad (3.2)$$

Where O_{jk} is the number of occurrences of the jth species in the kth community $k = (1, 2, \ldots, g)$.

The sorting method has five steps:.

1. An unsorted relevé table is created with groups (communities) in columns and species in rows. Responses within the table are fidelities of species to groups.
2. Groups (columns) are ordered with respect to some sort of meaningful gradient (e.g., soil moisture, soil depth, etc.).
3. Fidelities of species to groups are multiplied by a vector, v, of length g (where g is the number of groups). The vector is uniformly distributed from 1 to -1.
4. The vector v is multiplied element-wise by each row.

5. The species (rows) in the relevé table are sorted with respect to the sums of this multiplication, i.e., their dot product (Aho 2006b). One will obtain γ sums where γ = the number of species in the dataset.

A lack of environmental data hindered relevé sorting with respect to a meaningful order of columns. What appeared to be disturbed-type communities were placed on the far right side of tables. There appeared to be one such community in the 12-group classification (community 2), and two such communities in the 22-group classification (communities 10 and 13). These communities were dominated by either *Agropyron cristatum* or *Bromus tectorum* (note that many communities had a high abundance of *B. tectorum*). Other than this, communities (columns) were ordered left to right in the relevé table from least to greatest total community abundance (lowest to highest total ground cover of vegetation). Species (rows) were sorted with respect to this ordering of columns.

Conventional statistical summaries of 12- and 22-cluster classifications (Tables 3.1, 3.3) are included along with sorted relevés of the 12-and 22-cluster classifications (Tables 3.2, 3.4).

Table 3.1 Summary statistics for the 12-cluster classification.

Cluster	Plots	Total Richness	Plot Richness (mean ± SE)	Plot Cover (mean ± SE)	Simpson[1] Diversity (mean ± SE)	Shannon-Wein[2] Diversity (mean ± SE)	Beta[3] Diversity
1	21	51	11.3 ± 0.7	55.2 ± 2.4	0.68 ± 0.03	0.03 ± 0.1	3.5
2	22	48	7.8 ± 0.7	47.5 ± 1.9	0.5 ± 0.05	0.05 ± 0.1	5.2
3	26	54	10.3 ± 0.7	58.8 ± 2	0.62 ± 0.03	0.03 ± 0.1	4.2
4	53	68	10 ± 0.5	50.9 ± 1.3	0.65 ± 0.02	0.02 ± 0.1	5.8
5	28	66	11.9 ± 0.8	37.7 ± 1.8	0.74 ± 0.02	0.02 ± 0.1	4.5
6	41	74	11.3 ± 0.6	41.3 ± 1.7	0.71 ± 0.02	0.02 ± 0.1	5.5
7	30	59	10.1 ± 0.6	46.6 ± 2.5	0.68 ± 0.02	0.02 ± 0.1	4.9
8	16	55	14.2 ± 0.9	60.4 ± 3.3	0.76 ± 0.01	0.01 ± 0	2.9
9	26	57	11.2 ± 0.6	45.7 ± 2.5	0.65 ± 0.02	0.02 ± 0.1	4.1
10	16	57	12.6 ± 1.4	57.2 ± 3.9	0.69 ± 0.04	0.04 ± 0.1	3.5
11	28	63	9.7 ± 0.6	67 ± 4.2	0.62 ± 0.03	0.03 ± 0.1	5.5
12	7	14	4.9 ± 0.9	32.8 ± 2.8	0.54 ± 0.07	0.07 ± 0.2	1.9

[1] $D = 1 - \sum_i p_i^2$ (Simpson 1964), [2] $H' = \sum_i p_i \ln p_i$ (MacArthur and MacArthur 1961), where p_i is the proportion of species i in the sampling unit, [3] $\beta_W = (\gamma / \alpha) - 1$ (Whittaker 1960), where γ is the total number of species in the landscape, and α is average plot richness.

Table 3.2. Summary relevé table for 12 community types. This table lists all species that occur with >20% constancy in at least one community[1]. The two-character cipher[2] in each cell indicates the constancy and cover of the species in the community. Bolded cells indicate >30% constancy. Lightly shaded cells indicate 40%<constancy<70%. Dark shaded cells indicate ≥70% constancy.

	12	5	6	9	7	4	1	10	3	8	11	2
Atriplex falcata	**9D**	+A	+A	..	..	..	..	1A	++	..	..	+A
Halogeton glomeratus	**5C**	1A	+A	..	1A	1A	1A	+A	..	..	+A	1B
Krascheninnikovia lanata	**4B**	**5C**	2A	..	2A	+A	+A	**3A**	1A	..	..	1A
Artemisia arbuscula	..	2A	2C	+A	..	..	..	..	..	..	+A	..
Artemisia nova	..	1A	2C	1B	..	+A	+A	..	..	..	..	+A
Atriplex confertifolia	..	+A	**3C**	..	+A	+A	..	..	+A	..	..	+A
Eriogonum microthecum	1B	2A	2A	2A	+A	+A	++	1A	..	..	..	+A
Linanthus pungens	..	**3A**	**4B**	**3A**	2A	1A	..	1A	1A	1A	++	..
Tetradymia canescens	..	**3A**	1A	1A	+A	1A	+A	2A	..	1A	..	..
Astragalus calycosus	..	2A	+A	..	..	+A	..	2A	..	+A	..	..
Artemisia tridentata ssp. *wyo.*	..	2A	1A	**3B**	**9D**	**3B**	+A	+A	+A	+A	+A	2B
Pseudoroegneria spicata	..	+A	1A	**9D**	1B	**4D**	+A	+A	+A	**3A**	+A	+A
Juniperus osteosperma	..	..	..	2C	..	+A	..	..	+A	..	..	+A
Artemisia tripartita	..	..	..	2D	+A	+A	+A	..	..	+A	+A	..
Pascopyrum smithii	2A	**4D**	+A	..	..	+A	**3C**	1A	1A	1A	+A	+A
Poa secunda	..	2A	**8D**	**9D**	**5C**	**4B**	2A	2A	1A	**4C**	**3C**	**4B**
Opuntia polyacantha	..	**6A**	**4A**	1A	2A	**3A**	**4A**	**4A**	**5A**	2A	++	1A
Iva axillaris	..	**3B**	+A	..	..	+A	..	**3A**	..	..	1A	+A
Eriogonum ovalifolium	..	2A	+A	1A	+A	1A	..	1A	1A	..	..	1A
Arenaria franklinii	..	+A	1A	**3A**	+A	1A	+A	1A	+A	1A	+A	+A
Achnatherum hymenoides	**7C**	**8C**	**8C**	**4A**	**5B**	**4B**	**6B**	**9D**	**8C**	**8C**	**3B**	**3A**
Schoenocrambe linifolia	1A	**4A**	1A	++	**4A**	**3A**	**5A**	**4A**	+A	1A	1A	1A
Elymus elymoides	2A	**5A**	**6B**	**4A**	**9C**	**7C**	**5A**	**4B**	**4B**	**6A**	**3B**	**4B**
Chrysothamnus viscidiflorus	2A	**8D**	**7C**	**8C**	**8D**	**9D**	**7D**	**6C**	**6C**	**9D**	**4C**	**5C**
Eriastrum wilcoxii	..	1A	+A	+A	..	+A	2A	**3A**	++	1A	++	++
Phlox hoodii	..	**3A**	2A	**3A**	**4A**	**4A**	**3A**	1A	1A	**4A**	1A	**3A**
Descurainia pinnata	1A	1A	1A	**4A**	**3A**	**4A**	1A	1A	1A	**4A**	1A	**3A**
Eriogonum cernuum	..	1A	++	..	..	+A	+A	**3A**	1A	..	++	..
Chenopodium leptophyllum	..	1A	++	2A	+A	1A	2A	1A	1A	1A	1A	+A
Crepis acuminata	..	++	+A	**5A**	1A	1A	1A	2A	1A	**3A**	1A	1A
Erigeron pumilus	..	..	+A	1A	1A	1A	1A	+A	+A	2A	+A	+A

Table 3.2. Continued

	12	5	6	9	7	4	1	10	3	8	11	2
Artemisia tridentata ssp. *trid.*	..	2B	2C	..	++	+A	2B	1A	**3C**	1C	2A	1A
Cryptantha scoparia	..	+A	1A	+A	+A	+A	1A	1A	**3A**	1A	2A	..
Hesperostipa comata	..	**4B**	**3B**	1A	+A	2B	**3B**	**5D**	**9E**	**8D**	2B	+A
Machaeranthera canescens	..	1A	1A	1A	+A	+A	2A	1A	+A	**4A**	1A	++
Phlox longifolia	..	..	+A	2A	+A	+A	1A	..	..	2A	1A	+A
Mentzelia albicaulis	**4C**	**3A**	2A	+A	1A	+A	**5C**	**4A**	**7C**	1A	**4C**	1A
Pteryxia terebinthina	..	+A	+A	..	..	..	..	..	..	**3A**	..	..
Allium textile	..	..	+A	+A	++	+A	..	**3A**	..	1A	..	+A
Elymus lanceolatus	1A	**3B**	1A	1A	1A	**3B**	**9D**	**6C**	**7C**	**6C**	**3C**	1A
Psoralidium lanceolatum	..	+A	+A	..	..	+A	++	+A	**3A**	..	..	..
Bromus tectorum	..	1A	**3B**	**3A**	**3B**	**3A**	**4B**	2C	**3A**	**8C**	**8D**	**3A**
Alyssum desertorum	..	1A	2A	+A	2B	**3A**	**3B**	1A	2A	**9D**	**3C**	2A
Artemisia tridentata	..	1A	1A	..	2A	1A	..	+A	2A	+A	1A	**5D**
Tragopogon dubius	..	..	+A	1A	++	..	++	..	1A	2A	1A	..
Agropyron cristatum	1A	1A	1A	..	1A	1A	+A	+A	..	2C	1A	**6D**
Lappula occidentalis	..	++	+A	+A	++	1A	1A	1A	+A	1A	2A	+A
Salsola kali	..	+A	+A	+A	..	++	**3A**	**3A**	1A	..	**3B**	..
Descurainia sophia	..	+A	+A	..	+A	+A	1A	++	+A	+A	**4C**	+A
Sisymbrium altissimum	..	+A	+A	+A	..	+A	2A	2A	2A	1A	**5D**	+A
Leymus cinereus	..	..	..	..	+A	+A	..	..	..	+A	**4D**	1A

[1]Lower constancy species not included in table were: *Allium acuminatum, Antennaria microphylla, Arabis cobrensis, Arabis holboellii, Arabis lignifera, Artemisia ludoviciana, Astragalus convallarius, Astragalus curvicarpus, Astragalus filipes, Astragalus lentiginosus, Astragalus purshii, Castilleja angustifolia, Calochortus bruneaunis, Carex douglasii, Carduus nutans, Chaenactis douglasii, Chenopodium fremontii, Corallorhiza maculate, Cordylanthus ramosus, Cryptantha interrupta, Erysimum capitatum, Erodium cicutarium, Erigeron filifolius, Ericameria nauseosa, Ericameria nana, Gayophytum diffusum, Aliciella leptomeria, Gilia sinuata, Grayia spinosa, Gutierrezia sarothrae, Hedysarum boreale, Hordeum jubatum, Ionactis alpina, Ipomopsis congesta, Lactuca serriola, Langloisia setosissima, Leymus flavescens, Lomatium dissectum, Lomatium foeniculaceum, Lupinus argenteus, Lygodesmia grandiflora, Penstemon cyaneus, Penstemon radicosus, Phacelia glandulifera, Phacelia hastata, Populus angustifolia, Purshia tridentata, Ribes aureum, Rosa woodsii, Salix exigua, Sphaeralcea munroana, Stephanomeria spinosa, Stanleya viridiflora, Tetradymia spinosa, Thelypodium laciniatum, Tiquilia nuttallii, Townsendia florifer,* Unknown 1, Unknown 2, *Zigadenus venenosus.*

[2]For each cell in the body of the table, constancy is indicated by the first symbol, while cover is indicated by the second symbol. For constancy: 0% = ".", 0-10% = +, 10-20% = 1, 20-30% = 2, 30-40% = 3, 40-50% = 4, 50-60% = 5, 60-70% = 6, 70-80% = 7, 80-90% 8, 90-100% = 9. For cover: 0% = ".", 0-0.01% = +, 0.01-1% = A, 1-2% = B, 2-5% = C, 5-25% = D, >25% = E.

Table 3.3 Summary statistics for the 22-cluster classification.

cluster	plots	Total Richness	Plot Richness (mean ± SE)	Plot Cover (mean ± SE)	Simpson[1] Diversity (mean ± SE)	Shannon-Wein[2] Diversity (mean ± SE)	Beta[3] Diversity
1	21	51	11.3 ± 0.7	55.2 ± 2.4	0.68 ± 0.03	1.5 ± 0.1	3.5
2	9	35	9.6 ± 1.1	49.9 ± 2.4	0.65 ± 0.06	1.4 ± 0.2	2.7
3	26	54	10.3 ± 0.7	58.8 ± 2	0.62 ± 0.03	1.4 ± 0.1	4.2
4	53	68	10 ± 0.5	50.9 ± 1.3	0.65 ± 0.02	1.4 ± 0.1	5.8
5	22	61	12.7 ± 0.9	35.4 ± 1.6	0.78 ± 0.03	1.9 ± 0.1	3.8
6	10	48	13.2 ± 1	49.9 ± 2.5	0.77 ± 0.03	1.8 ± 0.1	2.6
7	30	59	10.1 ± 0.6	46.6 ± 2.5	0.68 ± 0.02	1.5 ± 0.1	4.9
8	16	55	14.2 ± 0.9	60.4 ± 3.3	0.76 ± 0.01	1.7 ± 0	2.9
9	6	29	9 ± 1	45.9 ± 4.7	0.61 ± 0.03	1.2 ± 0.1	2.2
10	13	38	6.5 ± 0.7	45.8 ± 2.8	0.4 ± 0.06	0.8 ± 0.1	4.8
11	21	54	10.7 ± 0.7	42.7 ± 2.7	0.65 ± 0.03	1.4 ± 0.1	4.1
12	16	57	12.6 ± 1.4	57.2 ± 3.9	0.69 ± 0.04	1.6 ± 0.1	3.5
13	11	39	9.7 ± 0.9	70.1 ± 3.8	0.64 ± 0.04	1.4 ± 0.1	3
14	7	31	9.6 ± 1.3	50 ± 4.5	0.48 ± 0.09	1.1 ± 0.2	2.2
15	7	14	4.9 ± 0.9	32.8 ± 2.8	0.54 ± 0.07	1 ± 0.2	1.9
16	14	52	11.9 ± 1.1	45.2 ± 2.4	0.73 ± 0.03	1.7 ± 0.1	3.4
17	10	40	9.8 ± 1.2	75.4 ± 9.5	0.7 ± 0.04	1.5 ± 0.1	3.1
18	5	28	13.4 ± 1.1	58.5 ± 3.3	0.63 ± 0.05	1.5 ± 0.1	1.1
19	7	23	10 ± 1	36 ± 2.6	0.74 ± 0.03	1.7 ± 0.1	1.3
20	3	17	9 ± 1.5	40.6 ± 5.3	0.54 ± 0.09	1.2 ± 0.2	0.9
21	4	21	12.5 ± 0.6	25 ± 2	0.83 ± 0.01	2 ± 0	0.7
22	3	13	6 ± 1.2	29 ± 1.1	0.36 ± 0.08	0.8 ± 0.2	1.2

[1] $D = 1 - \sum_i p_i^2$ (Simpson 1964), [2] $H' = \sum_i p_i \ln p_i$ (MacArthur and MacArthur 1961), where p_i is the proportion of species i in the sampling unit, [3] $\beta_W = (\gamma / \alpha) - 1$ (Whittaker 1960), where γ is the total number of species in the landscape, and α is average plot richness.

Table 3.4. Summary relevé table for 22 community types. This table lists all species that occur with >20% constancy in at least one community[1]. The two-character cipher[2] in each cell indicates the constancy and cover of the species in the community. Bolded cells indicate >30% constancy. Lightly shaded cells indicate 40%<constancy<70%. Dark shaded cells indicate ≥70% constancy.

	21	22	15	5	19	20	11	16	9	7	6	14	2	4	1	12	18	3	8	17	10	13
Ericameria nana	**9D**	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
Langloisia setosissima	2A	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
Gutierrezia sarothrae	**9A**	..	..	..	..	..	..	..	..	+A	+A	..	..	..	..	1A	..	..	..	..	..	..
Artemisia ludoviciana	..	**3A**	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	+A	..	..
Artemisia arbuscula	..	**3A**	..	2A	**9D**	..	+A	+A	..	..	..	1A	..	..	..	..	..	..	..	..	..	..
Arabis cobrensis	..	**3A**	..	..	..	..	..	+A	..	..	..	..	..	+A	..	..	..	..	..	..	..	..
Atriplex confertifolia	..	**9D**	..	+A	**7C**	**3A**	..	2B	..	+A	1B	..	1B	+A	..	..	..	+A	..	..	..	..
Artemisia nova	..	..	..	1A	1A	..	1B	**5D**	..	..	..	..	..	+A	+A	..	..	..	..	..	+A	..
Atriplex falcata	..	..	**9D**	+A	..	..	..	1A	..	..	..	..	..	..	..	1A	..	++	..	..	+A	..
Eriogonum microthecum	..	..	1B	**3A**	**5A**	..	1A	**4A**	..	+A	..	..	1A	+A	++	1A	**3A**	..	..	..	+A	..
Krascheninnikovia lanata	..	**3A**	**4B**	**7D**	**4B**	..	..	**4B**	..	2A	..	..	..	+A	+A	**3A**	..	1A	..	..	2A	..
Grayia spinosa	..	..	1B	1A	1A	**9D**	..	+A	..	+A	..	..	..	+A	1B	+A	..	++	..	+B	..	..
Astragalus calycosus	2A	..	..	2A	..	..	..	..	1A	..	+A	..	..	+A	..	2A	..	..	+A	..	..	..
Linanthus pungens	**7C**	..	..	**3B**	**7C**	..	1A	**4B**	1A	2A	1A	..	..	1A	..	1A	**9C**	1A	1A	..	..	+A
Ipomopsis congesta	**4A**	..	..	+A	1A	..	..	..	..	..	..	..	..	++	..	1A	..	..	+A	..	..	..
Juniperus osteosperma	..	..	..	..	..	..	2C	..	..	..	..	..	1A	+A	..	..	1A	+A	..	..	..	..
Tetradymia canescens	..	..	..	**4A**	1A	..	1A	2A	..	+A	1A	..	..	1A	+A	2A	1A	..	1A	..	..	..
Cordylanthus ramosus	..	..	..	..	..	..	1A	++	..	+A	+A	2A	1A	+A	..	..	1A	..	..	..	..	..
Halogeton glomeratus	..	**3A**	**5C**	2A	1A	..	..	+A	..	1A	..	..	..	1A	1A	+A	..	..	..	..	**3C**	1A
Artemisia tridentata ssp. *wyo.*	2A	..	..	**3A**	..	**3C**	1A	1A	..	**9D**	2B	..	2B	**3B**	+A	+A	**9D**	+A	+A	..	2A	+A

Table 3.4. Continued.

	21	22	15	5	19	20	11	16	9	7	6	14	2	4	1	12	18	3	8	17	10	13
Poa secunda	**9B**	..	..	2A	**9C**	**9C**	**9D**	**9D**	1A	**5C**	**7C**	1A	**4B**	**4B**	2A	2A	**9C**	1A	**4C**	+A	**4A**	**6D**
Opuntia polyacantha	**9A**	**3A**	..	**6A**	**7A**	**3A**	1A	**3A**	**3A**	2A	1A	..	2A	**3A**	**4A**	**4A**	1A	**5A**	2A	..	+A	1A
Pseudoroegneria spicata	..	..	..	+A	1A	..	**9D**	1A	..	1B	**4A**	..	2C	**4D**	+A	+A	**9D**	+A	**3A**	..	..	+A
Iva axillaris	..	..	..	2A	..	..	..	+A	**4C**	..	..	1A	..	+A	..	**3A**	..	..	..	2A	+A	..
Achnatherum hymenoides	**9B**	**9C**	**7C**	**9C**	**9C**	**3B**	**4A**	**9C**	**3B**	**5B**	**6C**	**4A**	**4A**	**4B**	**6B**	**9D**	1A	**8C**	**8C**	**4B**	2A	2B
Schoenocrambe linifolia	..	**3A**	1A	**4A**	..	..	++	2A	1A	**4A**	1A	2A	**3A**	**3A**	**5A**	**4A**	..	+A	1A	+A	+A	..
Pascopyrum smithii	..	..	2A	**3B**	..	..	..	1A	**9D**	..	+A	..	..	+A	**3C**	1A	..	1A	1A	1B	1A	..
Eriogonum ovalifolium	..	..	..	**3A**	..	..	+A	++	..	+A	+A	..	2A	1A	..	1A	**3A**	1A	..	..	+A	..
Chrysothamnus viscidiflorus	**4A**	..	2A	**9D**	**9B**	**6A**	**8C**	**6C**	**6B**	**8D**	**8D**	**7B**	**7D**	**9D**	**7D**	**6C**	**9C**	**6C**	**9D**	2B	**4B**	**5D**
Arenaria franklinii	..	..	..	+A	..	..	**3A**	**3A**	..	+A	+A	..	2A	1A	+A	1A	**3A**	+A	1A	..	..	1A
Elymus elymoides	..	**3A**	2A	**5A**	**9C**	**9B**	**3A**	**6A**	**4A**	**9C**	**7C**	2A	**7C**	**7C**	**5A**	**4B**	**9A**	**4B**	**6A**	+A	2A	**7C**
Chaenactis douglasii	..	..	..	1A	..	..	+A	++	1A	++	..	2A	..	+A	+A	..	..	++	1A	..	+A	+A
Phlox longifolia	..	..	..	..	..	..	**3A**	+A	..	+A	..	2A	1A	+A	1A	..	..	..	2A	..	+A	+A
Descurainia pinnata	2A	**3A**	1A	2A	..	**3A**	**4A**	+A	..	**3A**	**3A**	..	**4A**	**4A**	1A	1A	**5A**	1A	**4A**	1A	**3A**	+A
Phlox hoodii	**4A**	..	..	**3A**	..	..	**3A**	2A	**3A**	**4A**	**3A**	..	**5A**	**4A**	**3A**	1A	**5A**	1A	**4A**	+A	1A	**3A**
Artemisia tridentata	..	**6A**	..	1A	..	..	..	..	..	2A	**5A**	2B	**9D**	1A	..	+A	..	2A	+A	..	2B	+A
Allium acuminatum	..	..	..	..	..	..	..	..	..	..	..	..	2A	..	..	..	..	..	..	..	..	..
Erigeron pumilus	..	**3A**	..	..	..	..	1A	+A	..	1A	+A	..	2A	1A	1A	+A	1A	+A	2A	..	..	+A
Machaeranthera canescens	2A	..	..	1A	..	..	1A	1A	..	+A	1A	2A	..	+A	2A	1A	..	+A	**4A**	..	+A	2A
Eriogonum cernuum	..	..	..	1A	..	..	..	+A	1A	..	..	..	..	+A	+A	**3A**	..	1A	..	+A	..	..
Cryptantha scoparia	..	..	..	+A	..	**6A**	+A	2A	..	+A	2A	**4A**	..	+A	1A	1A	..	**3A**	1A	1A	..	+A
Eriastrum wilcoxii	2A	..	..	++	..	**3A**	+A	..	**6A**	..	+A	..	..	+A	2A	**3A**	..	++	1A	+A	+A	..

Table 3.4. Continued.

	21	22	15	5	19	20	11	16	9	7	6	14	2	4	1	12	18	3	8	17	10	13
Chenopodium leptophyllum	..	..	..	1A	..	..	2A	..	1A	+A	+A	1A	1A	1A	2A	1A	**3A**	1A	1A	+A	+A	+A
Crepis acuminata	..	..	..	..	..	..	**5A**	1A	1A	1A	1A	2A	1A	1A	1A	2A	**3A**	1A	**3A**	1A	1A	+A
Ericameria nauseosa	2A	..	..	+A	..	..	..	..	..	+A	1A	..	..	+A	..	1A	1A	+A	..	..	+A	+B
Artemisia tridentata ssp. trid.	..	..	..	1B	..	**6B**	..	..	**4C**	++	**8D**	**5C**	1A	+A	2B	1A	..	**3C**	1C	..	1A	1A
Mentzelia albicaulis	..	..	**4C**	2A	..	**6B**	+A	**4A**	**4B**	1A	2A	**5A**	1A	+A	**5C**	**4A**	..	**7C**	1A	**6D**	1A	2B
Hesperostipa comata	**9C**	..	..	**5B**	..	**3C**	1A	**4C**	1A	+A	**3C**	..	..	2B	**3B**	**5D**	..	**9E**	**8D**	2B	1A	2C
Allium textile	..	..	..	..	..	..	+A	++	..	++	+A	..	1A	+A	..	**3A**	..	..	1A	..	+A	..
Agropyron cristatum	2A	..	1A	1A	1A	**3B**	..	1A	1A	1A	1A	1A	**3C**	1A	+A	+A	..	..	2C	+A	**9E**	+A
Artemisia tripartita	..	..	..	..	..	..	+A	..	..	+A	..	..	..	+A	+A	..	**9E**	..	+A	..	..	+A
Bromus tectorum	**9C**	..	..	1A	..	..	**3A**	2A	1A	**3B**	**7B**	**5C**	**5B**	**3A**	**4B**	2C	..	**3A**	**8C**	**7D**	1A	**9E**
Tragopogon dubius	..	..	..	..	..	..	1A	2A	..	++	..	2A	..	..	++	..	..	1A	2A	+A	..	+A
Alyssum desertorum	**9B**	..	..	+A	1A	..	..	2A	1A	2B	**3B**	2C	**3A**	**3A**	**3B**	1A	**3A**	2A	**9D**	2B	1A	**5B**
Pteryxia terebinthina	2A	..	..	..	..	..	..	..	1A	..	1A	..	..	..	..	..	..	..	**3A**	..	..	..
Elymus lanceolatus	..	..	1A	**3B**	..	..	1A	2B	**4C**	1A	1A	1A	2B	**3B**	**9D**	**6C**	**3A**	**7C**	**6C**	**4D**	1A	2A
Psoralidium lanceolatum	..	..	..	+A	..	..	..	..	..	..	+A	..	..	+A	++	+A	..	**3A**	..	..	..	..
Lappula occidentalis	..	..	..	+A	..	**3A**	+A	..	..	++	+A	2A	1A	1A	1A	1A	..	+A	1A	1A	+A	**3A**
Descurainia sophia	..	..	..	+A	1A	..	..	+A	1A	+A	1A	**4B**	1A	+A	1A	++	..	+A	+A	**3B**	+A	**5C**
Chenopodium fremontii	..	..	..	..	..	**3A**	..	..	..	++	..	1A	1A	++	..	..	..	++	1A	2A	..	..
Leymus cinereus	..	..	..	..	..	..	..	..	..	+A	..	**9E**	1A	+A	..	..	..	..	+A	2A	1A	2C
Salsola kali	..	..	..	..	..	..	+A	+A	**3A**	..	..	..	..	++	**3A**	**3A**	..	1A	..	**5C**	..	2A
Salix exigua	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2A	..	..
Sisymbrium altissimum	..	..	..	..	..	..	+A	2B	1A	..	+A	..	..	+A	2A	2A	..	2A	1A	**8D**	+A	**6A**

[1]Lower constancy species not included in table were: *Antennaria microphylla, Arabis holboellii, Arabis lignifera, Astragalus convallarius, Astragalus curvicarpus, Astragalus filipes, Astragalus lentiginosus, Astragalus purshii, Carduus nutans, Castilleja angustifolia, Calochortus bruneaunis, Carex douglasii, Corallorhiza maculata, Cryptantha interrupta, Erysimum capitatum, Erodium cicutarium, Erigeron filifolius, Gayophytum diffusum, Aliciella leptomeria, Gilia sinuata, Hedysarum boreale, Hordeum jubatum, Ionactis alpina, Lactuca serriola, Leymus flavescens, Lomatium dissectum, Lomatium foeniculaceum, Lupinus argenteus, Lygodesmia grandiflora, Penstemon cyaneus, Penstemon radicosus, Phacelia glandulifera, Phacelia hastata, Populus angustifolia, Purshia tridentata, Ribes aureum, Rosa woodsii, Sphaeralcea munroana, Stephanomeria spinosa, Stanleya viridiflora, Tetradymia spinosa, Thelypodium laciniatum, Tiquilia nuttallii, Townsendia florifer,* Unknown 1, Unknown 2, *Zigadenus venenosus.*

[2]For each cell in the body of the table, constancy is indicated by the first symbol, while cover is indicated by the second symbol. For constancy: 0% = ".", 0-10% = +, 10-20% = 1, 20-30% = 2, 30-40% = 3, 40-50% = 4, 50-60% = 5, 60-70% = 6, 70-80% = 7, 80-90% 8, 90-100% = 9. For cover: 0% = ".", 0-0.01% = +, 0.01-1% = A, 1-2% = B, 2-5% = C, 5-25% = D, >25% = E.

STEP 4: REFINEMENT

During a meeting on 23 January 2009, three refinements were suggested by Stoller scientists to improve the 22-type classification created in step 3.

1. Community 11 should be subdivided into a legitimate number of smaller groups.
2. Distinctive classes with community 7 may have been obscured by the ubiquity of *Artemisia tridentata* var. *wyomingensis* (Table 3.4). As a result, it was suggested that community 7 be reanalyzed after dropping *A. tridentata* var. *wyomingensis.*
3. Similarly, the ubiquity of *Chrysothamnus viscidiflorus* may have obscured the capacity of the classification algorithm to distinguish classes within group 4 (Table 3.4). Thus it was suggested that community 4 be reanalyzed after dropping *C. viscidiflorus*.

It should be noted that communities 11, 7, and 4 had three of the four highest beta diversities of any of the identified communities from the 22-type classification (Table 3.3). In addition, community 4 was by far the largest group identified in the 22-type classification (53 sites; Table 3.3).

METHODS

Communities 4, 7, and 11 were separately reclassified using flexible beta linkage (β = -0.25) with Steinhaus dissimilarity (see Step 1). *Chrysothamnus viscidiflorus* data was intentionally excluded from the reclassification of community 4. *Artemisia tridentata* var. *wyomingensis* data was intentionally excluded from the reclassification of community 7. Classifications were each examined with six classification evaluators (see Step 2) to find an optimal number of clusters within each of these classifications.

RESULTS – CLASSIFICATION EVALUATION

Community 4 Reclassification

The non-geometric evaluators (i.e. ISA evaluators) and geometric evaluators (i.e. all other evaluators) were in disagreement with respect to optimal classification solutions for the community 4 reclassification. The non-geometric evaluators favored a 2-cluster solution (Fig. 4.1, Table 4.1), while the geometric evaluators favored a larger number of clusters (13-17) and perceived 2-cluster solutions as particularly poor (Fig. 4.1, Table 4.1). Considering all evaluators simultaneously, the average maximum was at 13 classes while the minimum was at two clusters (Fig. 4.2, Table 4.1). A small peak also occurred at six clusters (Fig. 4.2). Of additional concern is the problem of having sufficient sites for confident community description. The number of sites in classifications with two through 15 clusters is shown in Table 4.2. Note that all classification solutions with more than eight clusters all had at least one cluster with only two sites (Table 4.2).

Community 7 Reclassification

Like the 4 community reclassification, non-geometric and geometric evaluators were in disagreement with respect to the optimal number of clusters. Again, the non-geometric evaluators favored a 2-cluster solution (Fig. 4.3, Table 4.3), while the geometric evaluators favored a larger number of clusters (eight-10) and perceived 2-cluster solutions to be poor (Fig. 4.3, Table 4.3). Considering all evaluators simultaneously, the average maximum was at 10 classes, while the worst solution was four classes (Table 4.3, Fig. 4.4). Again, the number of sites within clusters is of concern (Table 4.4). All classifications with more than three clusters all had at least one cluster with only two sites.

Community 11 Reclassification

Non-geometric and geometric evaluators were in agreement with respect to the optimal number of communities for the community 11 reclassification. Both approaches indicate three clusters was a reasonable solution (Figs 4.5, 4.6, Table 4.5). These three clusters have between three and 12 sites.

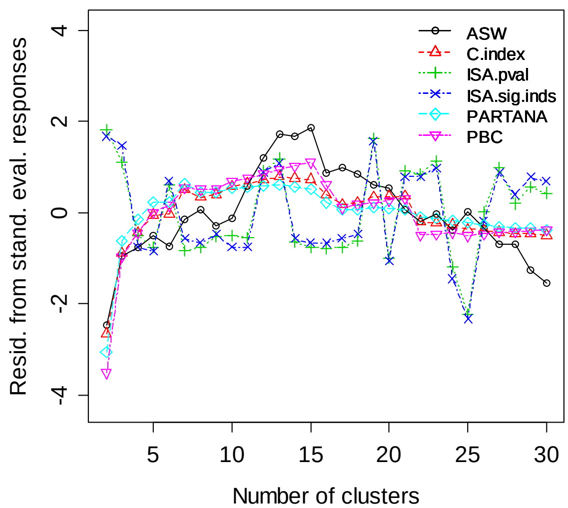


Figure 4.1. Scores of six evaluators with respect to 29 clustering solutions (two to 30 clusters) of sites within community 4. Classifications intentionally excluded *C. nauseosus*.

Table 4.1. Tabular summary of Figure 4.1

	best solution	worst solution
ASW	15	2
C-index	13	2
ISA avg. *p*-value	2	25
ISA # of sig. inds	2	25
PARTANA	17	2
PBC	15	2
Overall Avg.	13	2

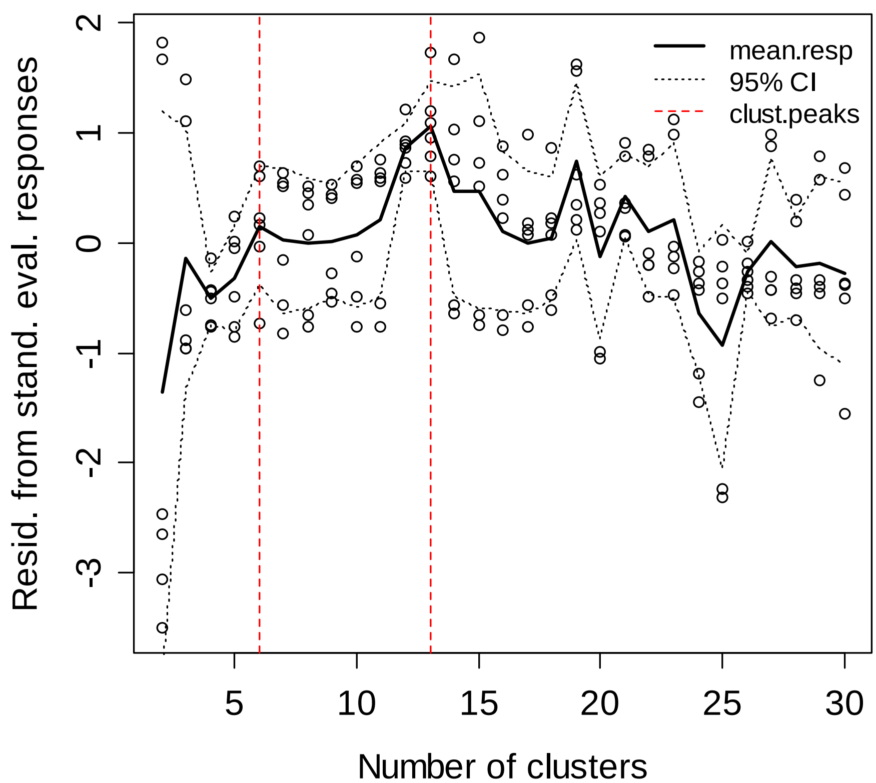


Figure 4.2. Scores of six evaluators with respect to 29 clustering solutions (two to 30 clusters) of a classification of community 4; *C. nauseosus* excluded. Averages are shown with a solid black line. Ninety-five percent confidence intervals around the mean shown with black dashed lines. Red dashed lines shown at 6 and 13 clusters, i.e. the maximum considering averages of all evaluators.

Table 4.2 Number of sites within clusters for 16 different classification solutions (two through17 clusters) for the community 4 reclassification. Cluster names are arbitrary.

	Number of clusters															
Cluster Name	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**
4a	34	19	16	16	8	8	5	5	5	5	5	5	5	5	5	5
4b	19	15	15	10	10	5	5	5	2	2	2	2	1	1	1	1
4c		19	19	5	5	5	5	5	5	5	5	3	1	1	1	1
4d			3	19	8	5	5	5	5	5	5	5	3	3	3	3
4e				3	19	8	8	8	8	6	3	3	5	5	5	5
4f					3	19	19	17	17	17	17	2	3	1	1	1
4g						3	3	3	3	3	3	17	2	2	2	2
4h							3	2	3	3	3	3	17	17	3	3
4i								3	2	2	3	3	3	2	14	11
4j									3	2	2	3	3	3	2	2
4k										3	2	2	3	3	3	3
4l											3	2	2	3	3	3
4m												3	2	2	3	3
4n													3	2	2	2
4o														3	2	2
4p															3	3
4q																3

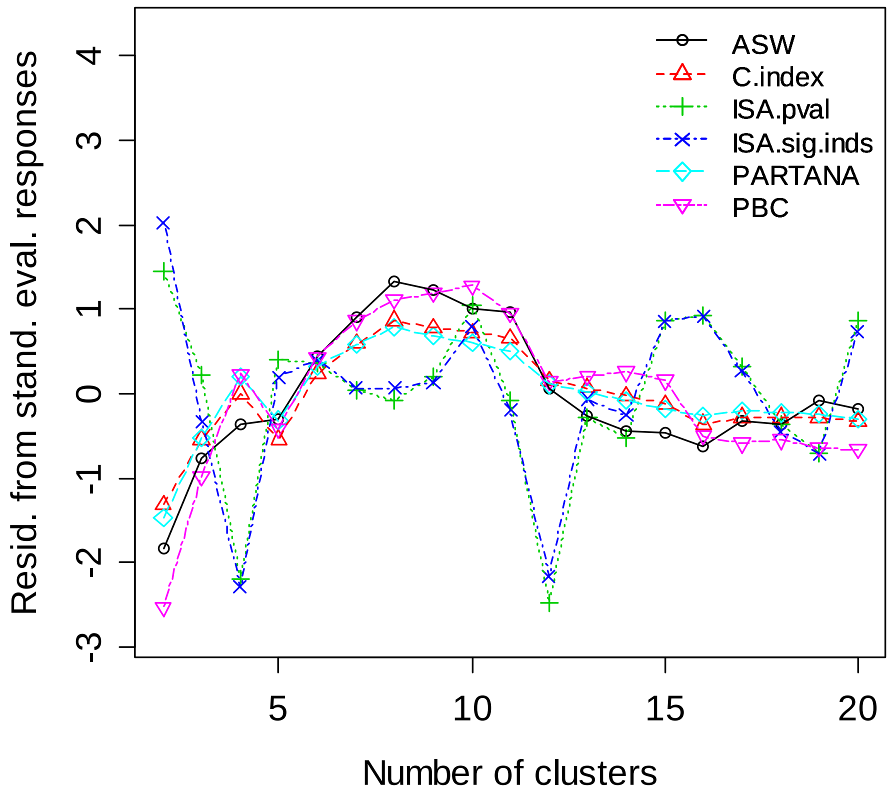


Figure 4.3. Scores of six evaluators with respect to 19 clustering solutions (two to 20 clusters) of sites within community 7. The classification excluded *A. tridentata* var *wyomingensis.*

Table 4.2. Tabular summary of Figure 4.3

	best solution	worst solution
ASW	8	2
C-index	8	2
ISA avg. *p*-value	2	12
ISA # of sig. inds	2	4
PARTANA	8	2
PBC	10	2
Overall Avg.	10	4

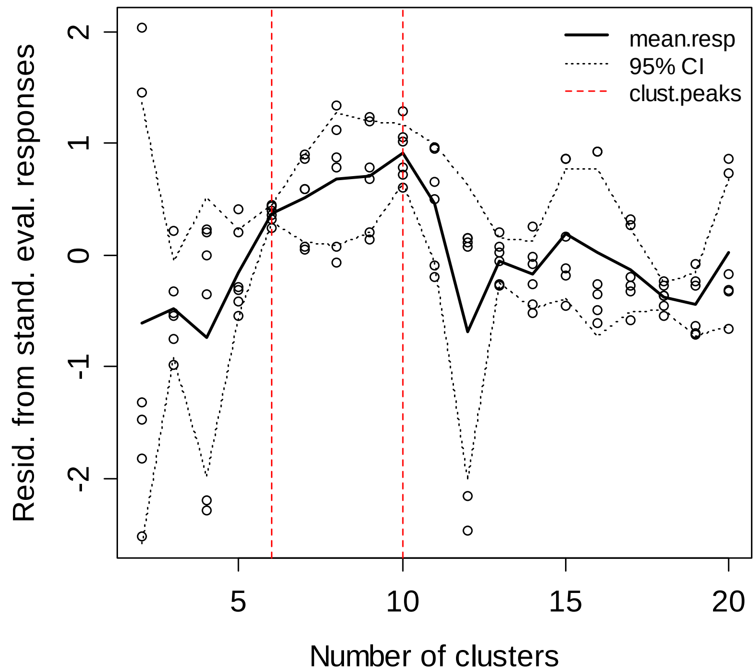


Figure 4.4. Scores of six evaluators with respect to 19 clustering solutions (two to 20 clusters) of a classification of community 7; *A. tridentata* var *wyomingensis* excluded. Averages are shown with a solid black line. Ninety-five percent confidence intervals around the mean shown with black dashed lines. Red dashed lines shown at six and 10 clusters.

Table 4.4 Number of sites for 10 different classification solutions for the community 7 reclassification (two through11 clusters). Cluster names are arbitrary.

		Number of clusters									
		2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Cluster name	**7a**	15	15	15	8	8	8	8	8	8	6
	7b	15	13	7	7	2	2	2	2	2	2
	7c		2	2	2	2	2	2	2	2	2
	7d			6	6	6	6	2	2	2	2
	7e				7	7	7	7	7	7	7
	7f					5	2	2	1	1	1
	7g						3	3	3	2	2
	7h							4	4	4	4
	7i								1	1	2
	7j									1	1
	7k										1

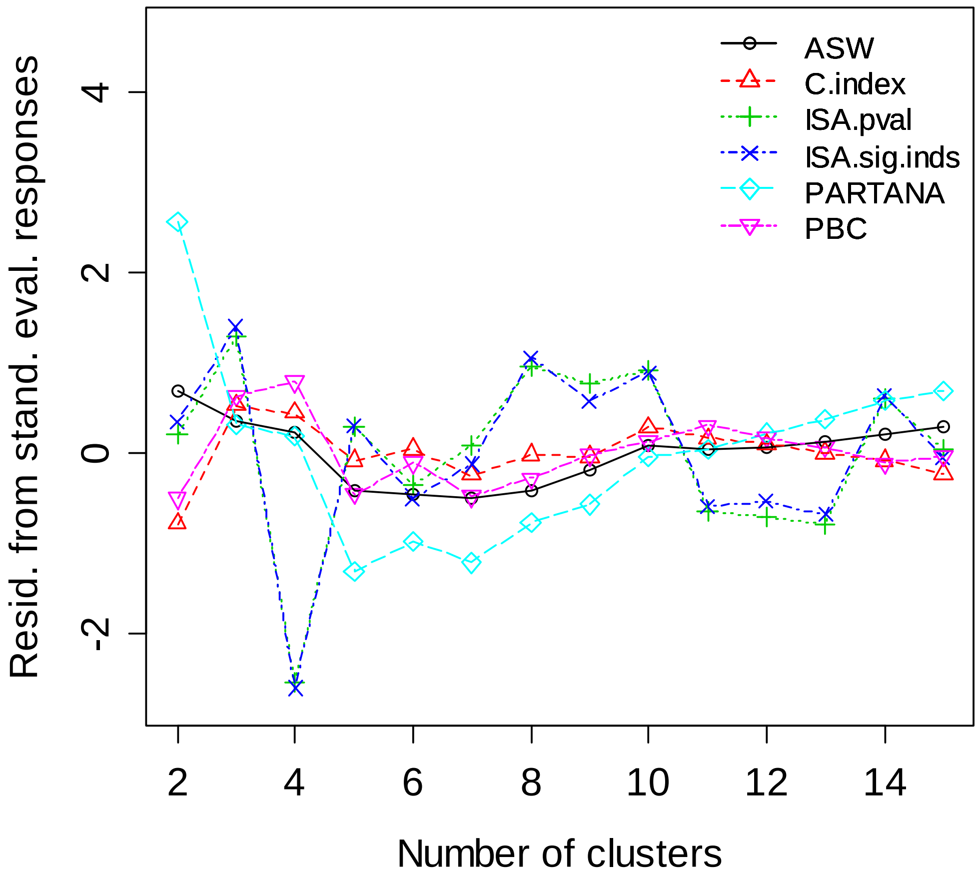


Figure 4.5. Scores of six evaluators with respect to 15 clustering solutions (two to 15 clusters) of sites within community 11.

Table 4.5. Tabular summary of Figure 4.5

	best solution	worst solution
ASW	2	7
C-index	3	2
ISA avg. *p*-value	3	4
ISA # of sig. inds	3	4
PARTANA	2	5
PBC	4	2
Overall Avg.	3	4

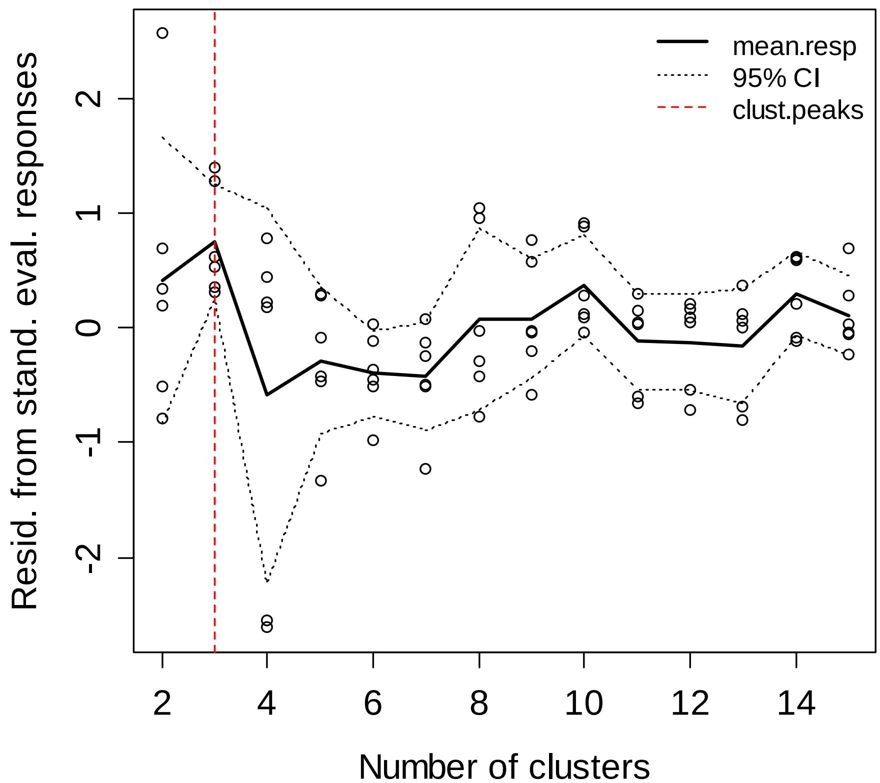


Figure 4.6. Scores of six evaluators with respect to 14 clustering solutions (two to 14 clusters) of a classification community 11. Averages shown with a solid black line. Ninety-five percent confidence intervals around the mean shown with black dashed lines. Red dashed line shown at three clusters.

STEP 5: UPDATED CLASSIFICATION

Classifications with two types for the community 4 and 7 reclassification and three types for the community 11 reclassification were chosen (see step 4). These are noted as communities 4a and 4b, 7a and 7b, and 11a, 11b, and 11c. Letting these types replace communities 4, 7, and 11 in the original 22 type classification results in 26 types. An updated classification with the community 4, 7, and 11 subtypes is described in this step (Tables 5.1, 5.2).

Table 5.1 Summary statistics for the updated 26-cluster classification

Cluster	Plots	Total Richness	Plot Richness (mean ± SE)	Plot Cover (mean ± SE)	Simpson[1] Diversity (mean ± SE)	Shannon-Wein[2] Diversity (mean ± SE)	Beta[3] Diversity
1	21	51	11.3 ± 0.7	55.2 ± 2.4	0.68 ± 0.03	1.5 ± 0.1	3.5
2	9	35	9.6 ± 1.1	49.9 ± 2.4	0.65 ± 0.06	1.4 ± 0.2	2.7
3	26	54	10.3 ± 0.7	58.8 ± 2	0.62 ± 0.03	1.4 ± 0.1	4.2
4a	34	59	9.2 ± 0.5	50.3 ± 1.7	0.63 ± 0.03	1.4 ± 0.1	5.4
4b	19	51	11.4 ± 0.9	52.1 ± 1.8	0.69 ± 0.03	1.5 ± 0.1	3.5
5	22	61	12.7 ± 0.9	35.4 ± 1.6	0.78 ± 0.03	1.9 ± 0.1	3.8
6	10	48	13.2 ± 1	49.9 ± 2.5	0.77 ± 0.03	1.8 ± 0.1	2.6
7a	15	38	9 ± 0.6	44.3 ± 3.5	0.64 ± 0.02	1.3 ± 0.1	3.2
7b	15	49	11.1 ± 1.1	49 ± 3.4	0.72 ± 0.02	1.6 ± 0.1	3.4
8	16	55	14.2 ± 0.9	60.4 ± 3.3	0.76 ± 0.01	1.7 ± 0.1	2.9
9	6	29	9 ± 1	45.9 ± 4.7	0.61 ± 0.03	1.2 ± 0.1	2.2
10	13	38	6.5 ± 0.7	45.8 ± 2.8	0.4 ± 0.06	0.8 ± 0.1	4.8
11a	6	24	9.3 ± 1.1	51.6 ± 6.3	0.54 ± 0.07	1.2 ± 0.2	1.6
11b	12	46	11.2 ± 1.1	38.8 ± 2.6	0.69 ± 0.02	1.5 ± 0.1	3.1
11c	3	21	11 ± 0.6	40.5 ± 6.2	0.73 ± 0.03	1.6 ± 0.1	0.9
12	16	57	12.6 ± 1.4	57.2 ± 3.9	0.69 ± 0.04	1.6 ± 0.1	3.5
13	11	39	9.7 ± 0.9	70.1 ± 3.8	0.64 ± 0.04	1.4 ± 0.1	3
14	7	31	9.6 ± 1.3	50 ± 4.5	0.48 ± 0.09	1.1 ± 0.2	2.2
15	7	14	4.9 ± 0.9	32.8 ± 2.8	0.54 ± 0.07	1 ± 0.2	1.9
16	14	52	11.9 ± 1.1	45.2 ± 2.4	0.73 ± 0.03	1.7 ± 0.1	3.4
17	10	40	9.8 ± 1.2	75.4 ± 9.5	0.7 ± 0.04	1.5 ± 0.1	3.1
18	5	28	13.4 ± 1.1	58.5 ± 3.3	0.63 ± 0.05	1.5 ± 0.1	1.1
19	7	23	10 ± 1	36 ± 2.6	0.74 ± 0.03	1.7 ± 0.1	1.3
20	3	17	9 ± 1.5	40.6 ± 5.3	0.54 ± 0.09	1.2 ± 0.2	0.9
21	4	21	12.5 ± 0.6	25 ± 2	0.83 ± 0.01	2 ± 0.1	0.7
22	3	13	6 ± 1.2	29 ± 1.1	0.36 ± 0.08	0.8 ± 0.2	1.2

[1] $D = 1 - \sum_i p_i^2$ (Simpson 1964), [2] $H' = \sum_i p_i \ln p_i$ (MacArthur and MacArthur 1961), where p_i is the proportion of species i in the sampling unit, [3] $\beta_W = (\gamma / \alpha) - 1$ (Whittaker 1960), where γ is the total number of species in the landscape, and α is average plot richness.

Table 5.2. Summary relevé table for 26 community types. This table lists all species that occur with >20% constancy in at least one community1. The two-character cipher2 in each cell indicates the constancy and cover of the species in the community. Bolded cells indicate >30% constancy. Lightly shaded cells indicate 40%<constancy<70%. Dark shaded cells indicate ≥70% constancy.

	21	22	15	5	19	20	11 a	11 b	11 c	16	9	7a	7b	6	14	2	4a	4b	1	12	18	3	8	17	10	13
Ericameria nana	9D	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
Langloisia setosissima	2A	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
Artemisia arbuscula	..	**3A**	..	2A	9D	..	**3B**	..	..	+A	..	..	..	..	1A	..	..	..	..	..	..	..	..	..	..	..
Calochortus bruneaunis	..	..	..	++	..	..	..	1A	**3A**	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
Hordeum jubatum	..	..	..	..	..	..	..	..	**3A**	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
Arabis cobrensis	..	**3A**	..	..	..	..	..	..	..	+A	..	..	..	..	..	..	+A	..	..	..	..	..	..	..	..	..
Atriplex confertifolia	..	9D	..	+A	7C	**3A**	..	..	..	2B	..	+A	+A	1B	..	1B	+A	..	..	..	..	+A	..	..	..	..
Artemisia nova	..	..	..	1A	1A	..	**4D**	..	..	**5D**	..	..	..	..	..	..	..	+A	+A	..	..	..	..	..	+A	..
Gutierrezia sarothrae	9A	..	..	..	..	..	..	..	..	..	..	..	+A	+A	..	..	..	..	..	1A	..	..	..	..	..	..
Atriplex falcate	..	..	9D	+A	..	..	..	..	..	1A	..	..	..	..	..	..	..	..	..	1A	..	++	..	..	+A	..
Eriogonum microthecum	..	..	1B	**3A**	**5A**	..	1A	2A	..	**4A**	..	..	1A	..	..	1A	+A	+A	++	1A	**3A**	..	..	..	+A	..
Krascheninnikovia lanata	..	**3A**	**4B**	7D	**4B**	..	..	..	..	**4B**	..	1A	2A	..	..	..	+A	+A	+A	**3A**	..	1A	..	..	2A	..
Arabis holboellii	..	..	..	+A	..	..	..	+A	..	..	..	..	..	1A	..	1A	..	..	..	..	1A	..	..	..	..	..
Grayia spinosa	..	..	1B	1A	1A	9D	..	..	..	+A	..	+A	..	..	..	..	+A	..	1B	+A	..	++	..	+B	..	..
Penstemon cyaneus	..	..	..	..	..	..	..	1A	..	..	..	..	..	..	..	..	..	..	..	..	1A	..	..	..	..	..
Astragalus calycosus	2A	..	..	2A	..	..	..	..	..	..	1A	..	..	+A	..	..	++	1A	..	2A	..	..	+A	..	..	..
Juniperus osteosperma	..	..	..	..	..	..	**3A**	+A	9D	..	..	..	..	..	..	1A	..	+A	..	..	1A	+A	..	..	..	..
Linanthus pungens	7C	..	..	**3B**	7C	..	..	**3A**	..	**4B**	1A	1A	**4A**	1A	..	..	+A	**3A**	..	1A	9C	1A	1A	..	..	+A
Ipomopsis congesta	**4A**	..	..	+A	1A	..	..	..	..	..	..	..	..	..	..	..	++	1A	..	1A	..	..	+A	..	..	..

Table 5.2. Continued

	21	22	15	5	19	20	11 a	11 b	11 c	16	9	7a	7b	6	14	2	4a	4b	1	12	18	3	8	17	10	13
Tetradymia canescens	..	..	..	4A	1A	..	..	2A	..	2A	..	..	1A	1A	..	..	+A	2A	+A	2A	1A	..	1A	..	..	..
Artemisia ludoviciana	..	3A	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	+A	..	..
Cordylanthus ramosus	..	..	..	..	..	..	4A	+A	..	++	..	..	+A	+A	2A	1A	++	1A	..	..	1A	..	..	..	..	..
Halogeton glomeratus	..	3A	5C	2A	1A	..	..	..	..	+A	..	1A	1A	..	..	..	2A	..	1A	+A	..	..	..	..	3C	1A
Poa secunda	9B	..	..	2A	9C	9C	9C	9D	6B	9D	1A	4A	7C	7C	1A	4B	3B	5B	2A	2A	9C	1A	4C	+A	4A	6D
Artemisia tridentata v. wyo	2A	..	..	3A	..	3C	..	2B	..	1A	..	9D	9D	2B	..	2B	3C	2B	+A	+A	9D	+A	+A	..	2A	+A
Opuntia polyacantha	9A	3A	..	6A	7A	3A	1A	..	6A	3A	3A	3A	+A	1A	..	2A	3A	2A	4A	4A	1A	5A	2A	..	+A	1A
Iva axillaris	..	..	..	2A	..	..	..	..	..	+A	4C	..	..	..	1A	..	+A	..	..	3A	..	..	..	2A	+A	..
Pseudoroegneria spicata	..	..	..	+A	1A	..	9E	9D	9D	1A	..	..	3C	4A	..	2C	1A	9D	+A	+A	9D	+A	3A	..	..	+A
Achnatherum hymenoides	9B	9C	7C	9C	9C	3B	6A	2A	9B	9C	3B	5A	5C	6C	4A	4A	5B	2B	6B	9D	1A	8C	8C	4B	2A	2B
Schoenocrambe linifolia	..	3A	1A	4A	..	..	..	..	3A	2A	1A	5A	3A	1A	2A	3A	3A	2A	5A	4A	..	+A	1A	+A	+A	..
Pascopyrum smithii	..	..	2A	3B	..	..	..	..	..	1A	9D	..	..	+A	..	..	+A	..	3C	1A	..	1A	1A	1B	1A	..
Eriogonum ovalifolium	..	..	..	3A	..	..	..	+A	..	++	..	..	1A	+A	..	2A	+A	2A	..	1A	3A	1A	..	..	+A	..
Arabis lignifera	..	..	..	..	..	..	..	..	3A	..	..	++	..	+A	..	..	..	++	..	..	..	++	..	..	..	..
Arenaria franklinii	..	..	..	+A	..	..	4A	2A	3A	3A	..	+A	+A	+A	..	2A	1A	2A	+A	1A	3A	+A	1A	..	..	1A
Chrysothamnus viscidiflorus	4A	..	2A	9D	9B	6A	8C	8C	6B	6C	6B	9D	8C	8D	7B	7D	9E	9D	7D	6C	9C	6C	9D	2B	4B	5D
Elymus elymoides	..	3A	2A	5A	9C	9B	3A	4A	..	6A	4A	9D	9B	7C	2A	7C	7C	5B	5A	4B	9A	4B	6A	+A	2A	7C
Descurainia pinnata	2A	3A	1A	2A	..	3A	4A	4A	6B	+A	..	3A	2A	3A	..	4A	4A	4A	1A	1A	5A	1A	4A	1A	3A	+A
Phlox longifolia	..	..	..	..	..	..	3A	3A	3A	+A	..	..	1A	..	2A	1A	+A	1A	1A	..	..	..	2A	..	+A	+A
Phlox hoodii	4A	..	..	3A	..	..	..	5A	..	2A	3A	5B	4A	3A	..	5A	2A	8B	3A	1A	5A	1A	4A	+A	1A	3A

Table 5.2. Continued

	21	22	15	5	19	20	11 a	11 b	11 c	16	9	7a	7b	6	14	2	4a	4b	1	12	18	3	8	17	10	13
Artemisia tridentate	..	6A	..	1A	..	..	..	..	..	..	..	2A	1A	5A	2B	9D	1A	1B	..	+A	..	2A	+A	..	2B	+A
Allium acuminatum	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2A	..	..	..	..	..	..	..	..	..	..
Erigeron pumilus	..	3A	..	..	..	..	1A	2A	..	+A	..	++	1A	+A	..	2A	1A	1A	1A	+A	1A	+A	2A	..	..	+A
Eriogonum cernuum	..	..	..	1A	..	..	..	..	..	+A	1A	..	..	..	..	..	+A	..	+A	3A	..	1A	..	+A	..	..
Machaeranthera canescens	2A	..	..	1A	..	..	1A	2A	..	1A	..	++	1A	1A	2A	..	..	1A	2A	1A	..	+A	4A	..	+A	2A
Astragalus purshii	..	..	..	++	..	..	..	+A	..	++	..	..	..	..	..	..	..	3A	..	..	..	..	++	..	..	+A
Chenopodium leptophyllum	..	..	..	1A	..	..	..	4A	..	..	1A	+A	++	+A	1A	1A	2A	1A	2A	1A	3A	1A	1A	+A	+A	+A
Crepis acuminate	..	..	..	..	..	..	4A	6A	3A	1A	1A	1A	1A	1A	2A	1A	+A	3A	1A	2A	3A	1A	3A	1A	1A	+A
Cryptantha scoparia	..	..	..	+A	..	6A	..	+A	3A	2A	..	1A	++	2A	4A	..	+A	1A	1A	1A	..	3A	1A	1A	..	+A
Ericameria nauseosa	2A	..	..	+A	..	..	..	..	..	..	..	+A	..	1A	..	..	+A	..	..	1A	1A	+A	..	..	+A	+B
Eriastrum wilcoxii	2A	..	..	++	..	3A	..	..	3A	..	6A	..	..	+A	..	..	+A	+A	2A	3A	..	++	1A	+A	+A	..
Townsendia florifer	..	..	..	+A	..	..	..	+A	..	+A	..	..	..	+A	..	..	++	++	+A	..	1A	..	1A	..	..	..
Artemisia tridentata v. *trid*	..	..	..	1B	..	6B	..	..	..	..	4C	..	+A	8D	5C	1A	1B	..	2B	1A	..	3C	1C	..	1A	1A
Mentzelia albicaulis	..	..	4C	2A	..	6B	..	+A	3A	4A	4B	++	1A	2A	5A	1A	1A	+A	5C	4A	..	7C	1A	6D	1A	2B
Hesperostipa comate	9C	..	..	5B	..	3C	1A	..	6D	4C	1A	..	1B	3C	..	..	2C	2A	3B	5D	..	9E	8D	2B	1A	2C
Agropyron cristatum	2A	..	1A	1A	1A	3B	..	..	..	1A	1A	1A	1A	1A	1A	3C	1A	..	+A	+A	..	..	2C	+A	9E	+A
Allium textile	..	..	..	..	..	..	1A	+A	..	++	..	..	++	+A	..	1A	+A	++	..	3A	..	..	1A	..	+A	..
Bromus tectorum	9C	..	..	1A	..	..	3A	3A	6B	2A	1A	1A	5C	7B	5C	5B	4B	2A	4B	2C	..	3A	8C	7D	1A	9E
Astragalus filipes	..	..	..	..	..	..	..	+A	3A	++	..	1A	1A	1A	..	1A	+A	2A	1A	+A	1A	1A	1A	..	..	+A
Tragopogon dubius	..	..	..	..	..	..	1A	1A	..	2A	..	..	++	..	2A	..	..	..	++	..	..	1A	2A	+A	..	+A

Table 5.2. Continued

	21	22	15	5	19	20	11a	11b	11c	16	9	7a	7b	6	14	2	4a	4b	1	12	18	3	8	17	10	13
Artemisia tripartita	..	..	..	..	..	..	1A	+A	..	..	..	..	1B	..	..	..	+A	1B	+A	..	**9E**	..	+A	..	..	+A
Pteryxia terebinthina	2A	..	..	..	..	..	..	..	..	..	1A	..	..	1A	..	..	..	..	..	..	..	..	**3A**	..	..	..
Alyssum desertorum	**9B**	..	..	+A	1A	..	..	..	..	2A	1A	1A	**3C**	**3B**	2C	**3A**	**3A**	2A	**3B**	1A	**3A**	2A	**9D**	2B	1A	**5B**
Elymus lanceolatus	..	..	1A	**3B**	..	..	..	1A	**3A**	2B	**4C**	+A	**3A**	1A	1A	2B	2A	**4C**	**9D**	**6C**	**3A**	**7C**	**6C**	**4D**	1A	2A
Psoralidium lanceolatum	..	..	..	+A	..	..	..	..	..	..	..	..	..	+A	..	..	+A	..	++	+A	..	**3A**	..	..	..	..
Lappula occidentalis	..	..	..	+A	..	**3A**	..	+A	..	..	..	..	++	+A	2A	1A	1A	1A	1A	1A	..	+A	1A	1A	+A	**3A**
Descurainia Sophia	..	..	..	+A	1A	..	..	..	..	+A	1A	1A	..	1A	**4B**	1A	+A	++	1A	++	..	+A	+A	**3B**	+A	**5C**
Lomatium foeniculaceum	..	..	..	..	..	..	..	..	..	..	..	..	+A	..	1A	..	..	2A	..	..	1A	..	1A	..	..	..
Chenopodium fremontii	..	..	..	..	..	**3A**	..	..	..	..	..	++	..	..	1A	1A	+A	..	..	..	..	++	1A	2A	..	..
Leymus cinereus	..	..	..	..	..	..	..	..	..	..	..	1A	..	..	**9E**	1A	+A	..	..	..	..	..	+A	2A	1A	2C
Salsola kali	..	..	..	..	..	..	..	+A	..	+A	**3A**	..	..	..	..	..	++	+A	**3A**	**3A**	..	1A	..	**5C**	..	2A
Sisymbrium altissimum	..	..	..	..	..	..	1A	+A	..	2B	1A	..	..	+A	..	..	+A	+A	2A	2A	..	2A	1A	**8D**	+A	**6A**
Lactuca serriola	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	++	..	..	..	..	1A	+A	..
Carduus nutans	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	1A	..	..
Salix exigua	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	2A	..	..

[1]Lower constancy species not included in table were: *Antennaria microphylla, Astragalus convallarius, Astragalus curvicarpus, Astragalus lentiginosus, Castilleja angustifolia, Carex douglasii, Corallorhiza maculata, Cryptantha interrupta, Erysimum capitatum, Erodium cicutarium, Erigeron filifolius, Gayophytum diffusum, Aliciella leptomeria, Gilia sinuata, Hedysarum boreale, Ionactis alpina, Leymus flavescens, Lomatium dissectum, Lupinus argenteus, Lygodesmia grandiflora, Penstemon radicosus, Phacelia glandulifera, Phacelia hastata, Populus angustifolia, Purshia tridentata, Ribes aureum, Rosa woodsii, Sphaeralcea munroana, Stephanomeria spinosa, Stanleya viridiflora, Tetradymia spinosa, Thelypodium laciniatum, Tiquilia nuttallii,* Unknown1, Unkown2, *Zigadenus venenosus.*

[2]For each cell in the body of the table, constancy is indicated by the first symbol, while cover is indicated by the second symbol. For constancy: 0% = ".", 0-10% = +, 10-20% = 1, 20-30% = 2, 30-40% = 3, 40-50% = 4, 50-60% = 5, 60-70% = 6, 70-80% = 7, 80-90% 8, 90-100% = 9. For cover: 0% = ".", 0-0.01% = +, 0.01-1% = A, 1-2% = B, 2-5% = C, 5-25% = D, >25% = E.

Literature Cited

Aho, K., Blicker, P., Neuman, D., and T. Weaver. 2005. Categorization and characterization of vegetation in relation to bond release criteria: CCV Phase IV characterization of pre-mine vegetation. Prepared for MDEQ and a consortium of coal resource extraction companies. Montana State University, Bozeman MT. 302 pgs.

Aho, K. 2006a. Multivariate clustering for objective classification of vegetation data. *In* 2006 Billings Land Reclamation Symposium, June 5-8, 2006, Billings MT . Jointly published by BLRS and ASMR 3134 Montavista Rd., Lexington KY 40502.

Aho, K. 2006b. Alpine and cliff ecosystems in the north-central Rocky Mountains. PhD dissertation, Montana State University. 408 pgs. http://etd.lib.montana.edu/etd/2006/aho/AhoK1206.pdf

Aho, K., Roberts, D. W., and T. Weaver. 2008. Using geometric and non-geometric internal evaluators to compare eight vegetation classification methods. Journal of Vegetation Science. 19: 549-562.

Beals, E. W. 1984. Bray-Curtis ordination: an effective strategy for analysis of multivariate ecological data. *Adv. Ecol. Res.* 14: 1-55.

Bray, R. J. & Curtis, J. T. 1957. An ordination of upland forest communities of Southern Wisconsin. *Ecol. Mong.* 27: 325-349.

Dale, M. B. 1991. Knowing when to stop: cluster concept=concept cluster. In Feoli, E. & L. Orloci (eds.), *Computer Assisted Vegetation Analysis*, Kluwer Academic Publishers, Dordrecht: 149-171.

Dufrêne M. & Legendre, P. 1997. Species assemblages and indicator species: the need for a flexible asymmetric approach. Ecol. Mong. 67(3): 345-367.

Hothorn, T., Hornik, K., van de Wiel, M. A., and A. Zeileis. 2008. Implementing a Class of Permutation Tests: The coin Package. Journal of Statistical Software 28(8), 1-23. http://www.jstatsoft.org/v28/i08/.

Hubert, L. J. & Levin, J. R. 1976. A general framework for assessing categorical clustering in free recall. Psychol. Bull. 83: 1072-1080.

Kaufman, L. & Rousseeuw, P. J. 1990. Finding groups in data, an introduction to cluster analysis. John Wiley & Sons, New York,US.

Kruskal, J. B., M. Wish. 1978. Multidimensional scaling. Sage Publications. Beverly Hills, CA. 93 pp.

Kutner, M. H., Nachtsheim, C. J., Neter, J., and W. Li. 2005. Applied linear statistical models, 5th edition. McGraw-Hill, Boston.

Lance, G. N. & Williams, W. T. 1967. A general theory of classification sorting strategies I. Hierarchical systems. Comp. J. 4: 373-380.

Maechler, M., Rousseeuw, P., Struyf, A., Hubert, M. 2005. Cluster analysis basics and extensions; unpublished.

MacQueen, J. 1967. Some methods for classification and analysis of multivariate observations. In L. M. Le Cam & J.

Neyman (eds.), Proceedings of the Fifth Berkeley Symposium on Mathematical Statistics and Probability, pp. 281–297. University of California Press. Berkeley, CA.

McCune, B. & Grace, J. B. 2002. Analysis of ecological communities. MjM Software design. Gelenden Beach OR, US.

McCune, B., and M. J. Mefford. 1999. PC-ORD ver. 4 multivariate analysis of ecological communities. MjM Software design. Gelenden Beach OR. 237 pp.

McGill, R., Tukey, J. W. & Larsen, W. A. 1978. Variations of box plots. Am. Stat. 32: 12-16.

McQuitty, L. L. 1960. Hierarchical linkage analysis for the isolation of types. Educ. Psychol. Meas. 20: 55-67.

Roberts, D. 2005. Vegetation classification in R, for labdsv ver. 1.1-1, vegetation ecology package. http://www.cran.r-project.org.

Rousseeuw, P. J. 1987. Silhouettes: A graphical aid to the interpretation and validation of cluster analysis. J. Comput. Appl. Math. 20: 53-65.

R Development Core Team. 2008. R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. ISBN 3-900051-07-0, http://www.R-project.org.

Sneath, P. H. A. 1957. The application of computers to taxonomy. J. Gen. Microbiol. 17: 201-226.

Sokal, R. R. & Michener, C. D. 1958. A statistical method for evaluating systematic relationships. University of Kansas Science Bulletin. 38: 1409-1438.

Ward, J. H., Jr. 1963. Hierarchical grouping to optimize an objective function. J. Am. Stat. Assoc. 58: 236-244.

Wishart, D. 1969. An algorithm for hierarchical classifications. Biometrics. 25: 165-170.